Progression in Multiplication Calculations
Structures of Multiplication (Haylock and Cockburn 2008)

Children should experience problems with all the different multiplication structures in a range of practical and relevant contexts e.g. money and measurement

Repeated addition
So many lots (sets) of so many
How many (how much) altogether
Per, each

Scaling
Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)

Commutative law
Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)

a \times b and b \times a are equal

4 \times 2 is the same as/equal to 2 \times 4
## End of Year Expectations

### Year 1
- Solve single step practical problems involving multiplication
- Use concrete objects, pictorial representations to explore grouping
- Make connections between arrays, number patterns and counting in twos, fives and tens
- Double numbers and quantities

### Year 2
- Understand multiplication as repeated addition
- Calculate mathematical statements for multiplication within the tables and write them using symbols
- Understand and solve problems involving arrays
- Ensure children understand that multiplication is commutative (can be done in any order)
- Understand that multiplication and division are inverse operations

## Possible concrete and visual representation

### Year 1
![Possible concrete and visual representation](image1)

- Count in twos, threes, fives from zero and tens from any number
- Emphasise number patterns
- Double number and quantities

### Year 2
![Possible concrete and visual representation](image2)

- Count in twos, threes, fives from zero and tens from any number
- Emphasise number patterns
- Introduction to multiplication tables. Practise to become fluent in multiplication facts for 2, 5 and 10

## Children’s Recording

### Year 1
- Practical only e.g. link to small world
- Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in
- Count in twos, fives and tens from different multiples
- Emphasise number patterns
- Double number and quantities

### Year 2
- Record practical work as number sentences
- Solve multiplication problems mentally
- 4 x 2 = 8
- 2 x 4 = 8

## Fluency

### Year 1
- Count in twos, fives and tens from different multiples
- Emphasise number patterns
- Double number and quantities

### Year 2
- Count in twos, threes, fives from zero and tens from any number
- Emphasise number patterns
- Introduction to multiplication tables. Practise to become fluent in multiplication facts for 2, 5 and 10
- Solve multiplication problems mentally
Multiplication – multiplication and division should be taught together – refer to structures of multiplication

**End of Year Expectations**

### Year 3
- Develop reliable written methods
- Understand and solve scaling problems
- Solve problems involving multiplication including correspondence

**Possible concrete and visual representation**

- **Cuisenaire to represent scaling**
- **Statue is 3 times as tall: 3 metres**
- **I am 1 metre tall**
- **Flexible array**
- **Arrays**
- **Place value counters**

**Teacher Modelling/Children’s Recording**

- **Children must use manipulatives alongside algorithms**

```
4 \times 13  \quad \text{‘four lots of thirteen’}
```

```
\[
\begin{array}{c}
4 \\
40 \\
\hline
10 \\
3
\end{array}
\]
```

- **Expanded methods – grid and area**

```
4 \times 10 \\
4 \times 3
```

```
40 + 12 = 52
```

- **Progressing to developing fluency in short multiplication**

```
1 \times 4 \\
5 \times 4
```

```
\[
\begin{array}{c}
1 \\
3 \\
\hline
3
\end{array}
\]
```

**Fluency**

- Count from 0 in multiples of 4, 8, 50 and 100
- Use multiples of 2, 3, 4, 5, 8, 10, 50 and 100
- Practise mental recall of multiplication tables – 3, 4 and 8x times tables
- Connect the 2, 4 and 8 times tables using doubling
- Develop efficient mental methods using commutativity and multiplication facts to derive related facts e.g. $4 \times 4 \times 12 = 12 \times 4 \times 5 = 12 \times 20$

**Year 4**

- Multiplying three numbers
- Solve two-step problems
- Multiplying by 0 and by 1
- Develop fluency in short multiplication using formal written layout
- Solve problems involving multiplication including using the distributive law, integer scaling problems and harder correspondence problems

**Possible concrete and visual representation**

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- **Arrays**
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**Teacher Modelling/Children’s Recording**

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```

**Fluency**

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall and use multiplication facts up to 12 x 12 with increasing fluency
- Derive multiplication facts with up to three-digits
- Recognise and use factor pairs and commutativity in mental calculations
- Use the distributive law
- Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations e.g. $2 \times 6 \times 5 = 10 \times 6$
## End of Year Expectations

### Year 5

- Multiply decimals with up to three decimal places
- Identify multiples and factors including finding all factor pairs of a number, and common factors of two numbers
- Solve problems involving all four operations where larger numbers are used by decomposing them into their factors
- Multiply whole numbers and those involving decimals by 10, 100 & 1000
- Understand and use multiplication and division as inverses including in problems involving missing numbers and balancing equations
- Solve problems involving multiplication and division including scaling by simple fractions
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime)
- Recognise and square and cube numbers and associated notation

### Year 6

- Multiply numbers up to 4-digit x TU
- Multiply numbers with up to two decimal places x whole number
- Multiply multi-digit numbers up to four-digits by a two-digit whole number
- Multiply single-digit numbers with up to two-decimal places by whole numbers
- Solve problems involving all four operations

## Possible concrete and visual representation

- **Cuisenaire to represent scaling**
- **Statue is 3 times as tall: 3 metres**
- **flexible array**
- **arrays**
- **place value counters**
- **bar models**

## Teacher Modelling/Children’s Recording

- **Children might use manipulatives alongside algorithms**

### Short multiplication

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### Long multiplication

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<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>4</td>
<td>4</td>
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<tr>
<td>x</td>
<td>2</td>
<td>6</td>
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## Fluency

- Count forwards in steps of powers of 10 from any given number up to 1,000,000
- Practise and extend use of formal written method of short multiplication
- Apply all multiplication tables frequently. Commit them to memory and use them confidently to make larger calculations
- Multiply numbers mentally drawing upon known facts
- Undertake mental calculations with increasingly large numbers
- Continue to use all multiplication tables to calculate mathematical statements in order to maintain fluency